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NON-DARCY FORCED CONVECTION THROUGH A CHANNEL ATTACHED TO AN OPEN CAVITY WITH NON-UNIFORM HEAT FLUX USING NANOFLUID

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Abstract- *A forced convection heat transfer phenomenon in a two-dimensional horizontal porous channel with an open cavity is investigated numerically. A non-uniform heat flux is located on the bottom surface of the cavity. Three different heating modes are considered at this wall. The rest of the surfaces are taken to be adiabatic. The physical domain is filled with water based nanofluid containing TiO_2 naparticles. The fluid enters from left and exits from right with initial velocity U_i and temperature T_i. Governing equations are discretized using the Penalty Finite Element Method. The simulation is carried out for a range of solid volume fraction ϕ (= 0%-15%) with Prandtl number Pr = 6.2. Results are presented in the form of streamlines, isothermal lines, average Nusselt number, mean temperature and mean velocity field for the mentioned parameter. Enhanced heat transfer rate is observed for increasing nanoparticle concentration ϕ.*

Keywords: Nanofluid, Forced convection, Channel with open cavity, Porous medium, Finite element method.

1. INTRODUCTION

Forced convection heat transfer in different geometries (for example, channel, pipe bend, channel with cavity) for porous media has many significant engineering applications; for example, geothermal engineering, solar-collectors, performance of cold storage, and thermal insulation of buildings. Nanofluid technology has emerged as a new enhanced heat transfer technique in recent years. Nanofluid is made by adding nanoparticles and a surfactant into a base fluid can greatly enhance thermal conductivity and convective heat transfer. The diameters of nanoparticles are usually less than 100 nm which improves their suspension properties.

A considerable number of published articles are available that deal with flow characteristics, heat transfer, flow and heat transfer instability, transition to turbulence, design aspects, etc. Aminossadati and Ghasemi [1] and Manca et al. [2] made study on mixed convection in a channel with an open cavity. Significant contributions have been made by several researchers [3–6] in order to model the problems of cavities filled with porous medium that obeys the Darcy law. Kumar et al. [7] found the significant heat transfer enhancement by the dispersion of nanoparticles in the base fluid. The possible determining factors for the heat transfer reduction in nanofluids include the variations of the size, shape, and distribution of nanoparticles and uncertainties in the thermophysical properties of nanofluids. Most of the published papers are concerned with the analysis of natural convection heat transfer of nanofluids in square or rectangular enclosures; for example, Muthtamilselvan et al. [8] and Abu-Nada et al. [9].

In reality forced convection in a differentially heated enclosure is a prototype of many industrial applications and has received considerable attention because of its applicability in various fields. The study of convective flow in a complicated geometry is more difficult than that of square or rectangular enclosures.

Many studies have described the larruping behaviors of nanofluids, such as their effective thermal conductivity under static conditions on the convective heat transfer associated with fluid flow phenomena. Chen et al. [10] investigated convective flow drag and heat transfer of CuO nanofluid in a small tube. Their results showed that the pressure drop of the nanofluid per unit length was greater than that of water. The pressure drop increased with the increasing weight concentration of nanoparticles. In the laminar flow region, the pressure drop had a linear relationship with the *Re* number, while in the turbulent flow region, the pressure drop increased sharply with the increase of the *Re* number. The critical *Re* number became lower while the tube diameter was smaller. The convective heat transfer was obviously enhanced by adding nanoparticles. The nanoparticle weight concentration and flow status were the main factors influencing the heat transfer coefficient: the heat transfer coefficient increased with the increasing weight concentration, and the enhancement of heat transfer in the turbulent flow region was greater than that in the laminar flow region.

Parvin et al. [11] studied thermal conductivity variation on natural convection flow of water-alumina nanofluid in an annulus. It was found that significant heat transfer enhancement could be obtained due to the presence of nanoparticles and that this was accentuated by increasing the nanoparticles volume fraction and Prandtl number at moderate and large Grashof number using both models. Nasrin et al. [12] conducted transient analysis on forced convection phenomena in a fluid valve using nanofluid. They concluded that the rate of heat transfer in the fluid valve reduced for longer time periods.

Up to now, all the nanofluids in the existing researches used a surfactant to help nanoparticle suspending. Due to the stickiness of the surfactant, the sedimentation on the tube wall became colloid and was difficult to clear. The nanoparticles in the sedimentation could not suspend again. This kind of nanofluid would jam pipes during a prolonged period of operation. In order to overcome this disadvantage, our study used TiO_2 nanoparticle suspensions (consisting of de-ionized water and nanoparticles without surfactant). During the flow process, the good suspension properties remained. So far, there is no research on the analysis of forced convective heat transfer on nanofluid in a channel. Therefore, forced convection also becomes a crucial point of this study. The main issues discussed in this paper are: the convective flow and heat transfer characteristics of water-TiO_2 nanofluid in a horizontal channel with an open cavity. The main issue discussed in this paper is the effects of solid volume fraction on forced convective flow and heat transfer characteristics of water-TiO_2 nanofluid through a channel with an open cavity.

2. PHYSICAL CONFIGURATION

Fig. 1 shows a schematic diagram of the horizontal channel with an open cavity. The model describes a channel with two insulated walls. Flow enters from left and leaves from the right. The open cavity lies at the lower wall of the channel. The vertical walls of the cavity are considered as adiabatic and the bottom surface is heated with non uniform heat flux. The inlet fluid velocity and temperature are U_i and T_i respectively. The working fluid through the channel with open cavity is water-TiO_2 nanofluid. The thermo-physical properties of fluid (water) and solid TiO_2 are taken from [13].

3. MATHEMATICAL FORMULATION

In the present problem, it is considered that the flow is steady, two-dimensional, laminar, incompressible and there is no viscous dissipation**.** The radiation effect is neglected. The dimensionless governing equations under Boussinesq approximation are as follows

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial X}+Pr\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{\nu_{nf}}{Da}U$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y}+Pr\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\nu_{nf}}{Da}V$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)$$

where, $\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$ is the density, $(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s$ is the heat capacitance, $\beta_{nf}=(1-\phi)\beta_f+\phi\beta_s$ is the thermal expansion coefficient, $\alpha_{nf}=k_{nf}/(\rho C_p)_{nf}$ is the thermal diffusivity, the viscosity of the nanofluid is considered by the Pak and Cho correlation [14]. This correlation is given as

$$\mu_{nf}=\mu_f\left(1+39.11\phi+533.9\phi^2\right)$$

The effective thermal conductivity of the nanofluid is approximated by the Maxwell-Garnett model [15]:

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}$$

The corresponding boundary conditions are:
at the inlet: $\theta=0$, $U=1$
at the outlet: convective boundary condition $P=0$
at the bottom wall: $\frac{\partial \theta}{\partial Y}=-1$ at $0.1\le X\le 0.15$, $\frac{\partial \theta}{\partial Y}=-2$ at $0.15\le X\le 0.25$ and $\frac{\partial \theta}{\partial Y}=-1$ at $0.25\le X\le 0.3$

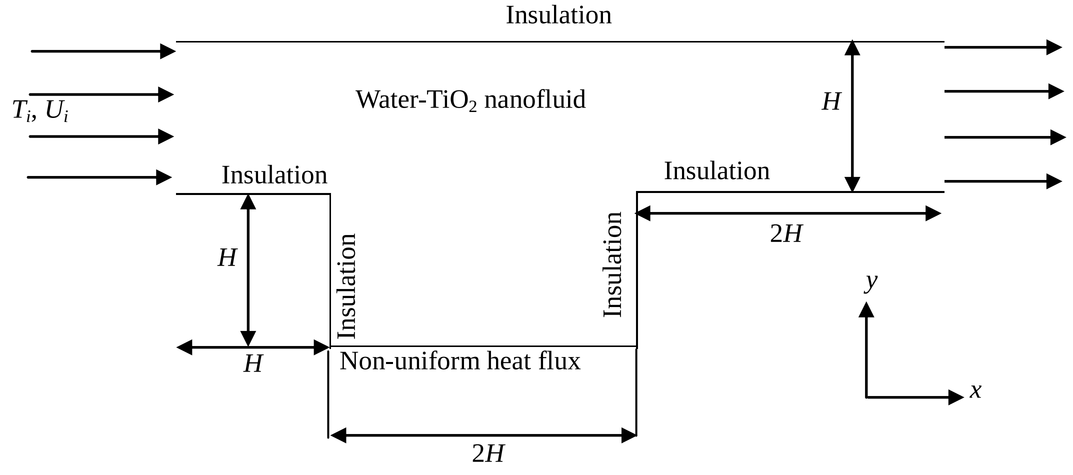


Fig. 1: Depiction of the geometry and the operation of the channel with an open cavity

at the vertical walls of the cavity: $U = V = 0$ and $\frac{\partial \theta}{\partial X} = 0$

at the horizontal boundaries of the channel: $U = V = 0$ and $\frac{\partial \theta}{\partial Y} = 0$

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables

$Pr = \frac{\nu_f}{\alpha_f}$ is Prandtl number, $Re = \frac{U_i H}{\nu_f}$ is Reynolds number and $Da = \frac{K}{H^2}$ is Darcy number.

The average Nusselt numbers at the bottom heated surface of the enclosure may be expressed as

$$Nu = -\frac{1}{L_S}\int_0^{L_S}\left(\frac{k_{nf}}{k_f}\right)\frac{\partial \theta}{\partial Y}dX \cdot$$

For convenience, a normalized average Nusselt number is defined as the ratio of the average Nusselt number at any volume fraction of nanoparticles to that of the pure water, which is:

$$X = \frac{x}{H},\ Y = \frac{y}{H}, U = \frac{u}{U_i}, V = \frac{v}{U_i},\ P = \frac{p}{\rho_f U_i^2},\ \theta = \frac{(T - T_i)k_f}{qH}$$

$$Nu^*(\phi) = \frac{Nu(\phi)}{Nu(\phi = 0)} \cdot$$

The mean temperature and velocity field of the nanofluid inside the domain are $\theta_{av} = \int \theta \, d\bar{V} / \bar{V}$ and $\omega_{av} = \int \omega \, d\bar{V} / \bar{V}$ respectively.

Here L_S and $\bar{V}$ are the non-dimensional length of the cavity bottom surface and volume of the channel with open cavity respectively.

4. NUMERICAL TECHNIQUE

The penalty finite element method [16] is used to solve the Eqs. (2) - (4), where the pressure *P* is eliminated by a penalty constraint. The continuity equation is automatically fulfilled for large values of this penalty constraint. Then the velocity components (*U*, *V*), temperature (θ) and concentration (*C*) are expanded using a basis set. The Galerkin finite element technique yields the subsequent nonlinear residual equations. Three points Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\left|\psi^{n+1} - \psi^{n}\right| \le 10^{-4}$, where n is the number of iteration and Ψ is a function of *U*, *V*, θ and *C*.

4.1 Mesh Generation

In finite element method, the mesh generation is the technique to subdivide a domain into a set of sub-domains, called finite elements, control volume etc. The discrete locations are defined by the numerical grid, at which the variables are to be calculated. It is basically a discrete representation of the geometric domain on which the problem is to be solved. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Fig. 2 displays the finite element mesh of the present physical domain.

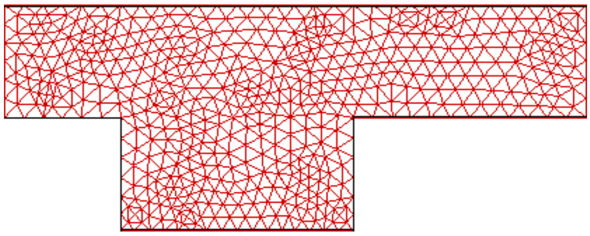


Fig. 2: Mesh generation of the channel with open cavity

4.2 Grid Refinement Test

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for *Re* = 10, *Pr* = 6.2, *Da* = 0.1 and ϕ = 5%. The extreme value of *Nu* is used as a sensitivity measure of the accuracy of the solution and is selected as the monitoring variable. Considering both the accuracy of numerical value and computational time, the present calculations are performed with 12666 nodes and 9059 elements grid system. This is described in Table 1.

Table 1. Grid Sensitivity Check at *Re* = 10, *Pr* = 6.2, *Da* = 0.1 and ϕ = 5%

Nodes (elements)	3224 (2864)	5982 (4930)	8538 (7014)	12666 (9059)	20524 (11426)
Nu	3.51465	5.40132	6.00158	6.41014	6.41015
Time (s)	259.265	392.594	488.157	521.328	727.375

4.3 Code Validation

The model validation is an important part of a numerical investigation. Hence, the outcome of the present numerical code is benchmarked against the numerical result of Mashaei et al. [17] which was reported for numerical investigation of nanofluid forced convection in channels with discrete heat sources. The comparison is conducted for the profile of thermal-hydraulic performance (η)-Reynolds number (*Re*) employing two different values of the solid volume fraction (ϕ). Fig. 3 executes an excellent agreement with Mashaei et al. [17]. This figure expresses the effect of *Re* and ϕ on η. The validation boosts the confidence to carry on the above stated objective of the current investigation.

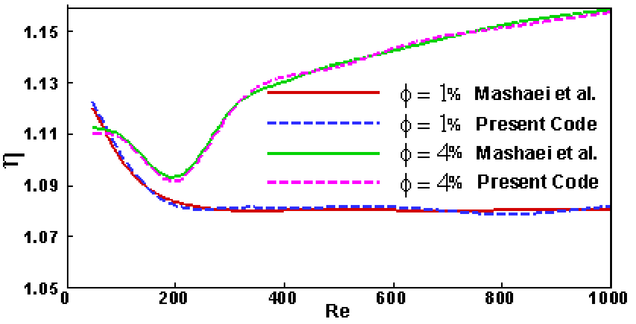


Fig. 3: Comparison of present code with Mashaei et al.[17]

5. RESULTS AND DISCUSSION

In this section, numerical results in terms of isotherms and streamlines are displayed for various solid volume fraction ϕ (= 0%, 5%, 10% and 15%) while Re = 10, Pr = 6.2 and and Da = 0.1 are kept fixed. In addition, the values of the local, average and normalized Nusselt number, mean bulk temperature and sub domain mean velocity profile through the channel with an open cavity have been calculated for water-TiO_2 nanofluid.

5. 1 Effect of solid volume fraction

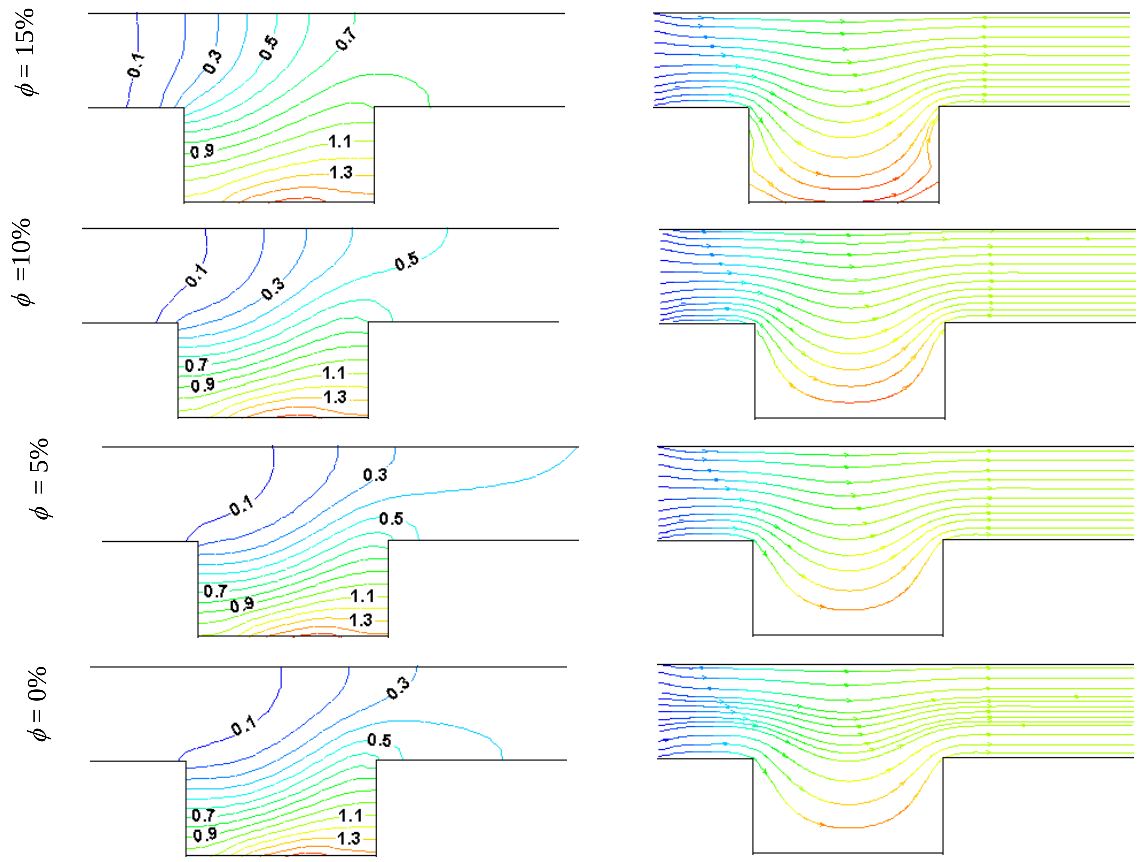
Figs. 4 (a) –4 (b) show the influences of solid volume fraction (ϕ) on the temperature and velocity profiles while Pr = 6.2. The strength of the thermal current and flow circulation activities is much more activated with escalating ϕ. The temperature lines through the horizontal pipe with an open cavity dense near the inlet for increasing ϕ. But initially (ϕ = 0%) these lines try to gather near the heated bottom wall of the cavity corresponding to physical changes of the working fluid. Increasing solid volume fraction causes the enhancement of thermal conductivity of the nanofluid. Due to rising values of ϕ, the temperature distributions become distorted resulting in an increase in the overall heat transfer. This result can be attributed to the performance of the solid volume fraction. It is worth noting that as the ϕ increases, the thickness of the thermal boundary layer near the input opening enhances which indicates a steep temperature gradient and hence, an increase in the overall heat transfer through the channel with an open cavity. In the velocity vector, initially the flow concentrates near the middle part of the channel while it covers the whole domain of the channel due to increase solid volume fraction from 0% to 15%. This is due to the fact that base fluid (ϕ = 0%) moves rapidly than solid concentrated water-TiO_2 nanofluid.

5.2 Local Nusselt number

A plot of the local Nusselt number (Nu_{local}) along the bottom heated wall of the cavity for different ϕ is expressed in Fig. 5. The shape of Nu_{local} – X (0.1-0.3) profile is parabolic. Local heat transfer increases for rising the solid volume fraction of the water-TiO_2 nanofluid.

5.3 Mean Nusselt number

Fig. 6 depicts the mean Nusselt number (Nu) with the variation of ϕ. In this figure we consider heat transfer rate



(a) (b)

Fig. 4: Effect of ϕ on (a) isotherms and (b) streamlines at Pr = 6.2

as a function of Reynolds number (*Re*). We consider the values of Reynolds number are 10, 100, 300 and 500. From *Nu*-*Re* profiles for solid volume fraction it is clearly shown that escalating ϕ lead the enhancement of heat transfer rate. Rate of heat transfer rises by 13% with the increasing values ϕ from 0% to 15% respectively.

5.4 Average temperature

The mean bulk non-dimensional temperature (θ_{av}) of the nanofluid inside the channel with an open cavity for the effect ϕ is displayed in Fig. 7. Average temperature θ_{av} of the fluid grows up with the variation of ϕ. This is due to the fact that thermal conductivity of nanofluid is always higher than base fluid ($\phi = 0\%$).

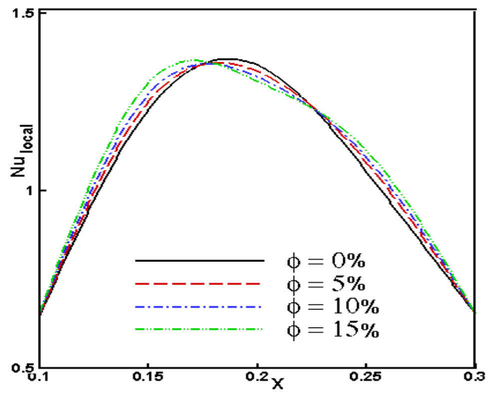


Fig. 5: local Nusselt number for the effect of ϕ

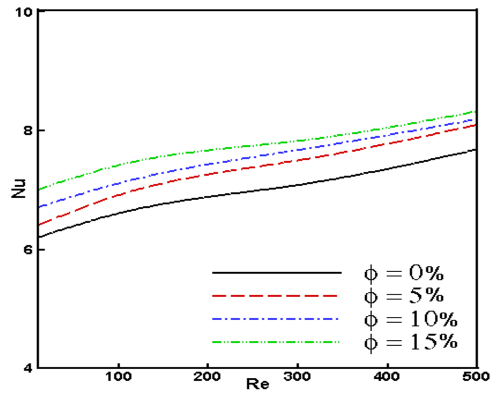


Fig. 6: Plot of average Nusselt number for various ϕ

5.5 Mean velocity

Fig. 8 states the mean sub domain velocity (ω_{av}) inside the channel with an open cavity for various ϕ. From the figure, it is found that a significant variation in average velocity field is found due to changing the acting parameter. ω_{av} grows down gradually with the escalating solid volume fraction.

5.6 Normalized Nusselt number

The design of normalized Nusselt number (*Nu**) for the effect of solid volume fraction ϕ is represented in Fig. 9. *Nu** grows with the variation of solid volume fraction for the water based TiO_2 nanofluid.

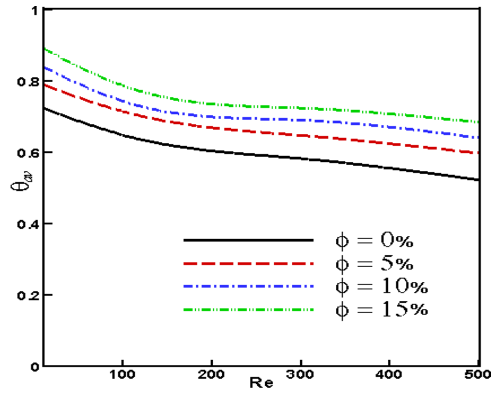


Fig. 7: Mean temperature of nanofluid for ϕ

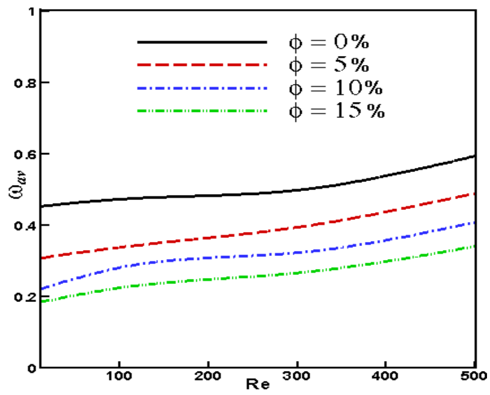


Fig.8: Mean velocity profile for ϕ

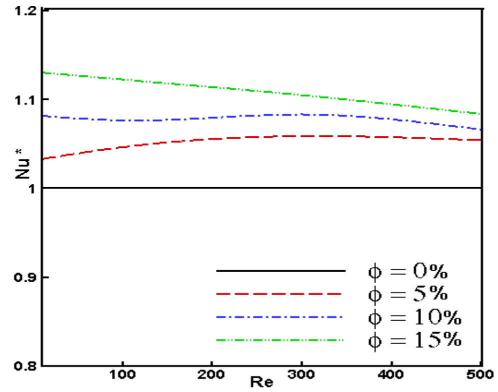


Fig. 9: Normalized Nusselt number

6. CONCLUSION

The problem of the effects of Prandtl number and solid volume fraction on forced convection heat transfer in a horizontal channel with an open cavity filled with water-TiO_2 nanofluid has been studied numerically. Flow and temperature field in terms of streamlines and isotherms have been displayed. The results of the numerical analysis lead to the following conclusions:

- The structure of the fluid flow and temperature field through the channel is found to be significantly dependent upon the solid volume fraction.
- There is a considerable change in the distribution of local Nusselt number.
- The maximum rate of heat transfer is obtained for the highest value of ϕ.

- The mean temperature of the fluid in the channel decrease with decreasing ϕ.

7. ACKOWLEDGEMENT

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9. NOMENCLATURE

Symbol	Meaning	Unit
C_p	Specific heat at constant pressure	(kJ kg^{-1} K^{-1})
Da	Darcy number State	Dimentio-nless
H	height of channel	(m)
k	Thermal conductivity	(W m^{-1} K^{-1})
Nu	Nusselt number	Dimentio-nless
Nu^*	Normalized Nusselt number	Dimentio-nless
p	pressure	(Kg m^{-1} s^{-2})
P	Non-dimensional pressure	Dimentio-nless
Pr	Prandtl number	Dimentio-nless
q	Heat flux,	(W m^{-2})
Re	Reynolds number	Dimentio-nless
T	temperature	(^{o}K)
u, v	x and y components of velocity	(m s^{-1})
U, V	Dimensionless velocities	Dimentio-nless
X, Y	Dimensionless coordinates	Dimentio-nless
x, y	coordinates	(m)
α	Fluid thermal diffusivity	(m^2 s^{-1})
β	Thermal expansion coefficient	(K^{-1})
ϕ	Nanoparticles volume fraction	Dimentio-nless
ν	Kinematic viscosity	(m^2 s^{-1})
θ	Dimensionless temperature	Dimentio-nless
ρ	Density	(kg m^{-3})
μ	Dynamic viscosity	(N s m^{-2})
ω	Dimensionless velocity field	Dimentio-nless