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OPTIMIZATION OF EFFECTIVENESS FOR A CYLINDRICAL FIN

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Abstract- *A numerical study was performed to provide information about the temperature distribution of three dimensional cylindrical fin in steady state and homogeneous material properties. A brief literature review shows that much of work on fins has been carried out analytically and numerically in one dimensional and two dimensional conditions. This study is concerned about the three dimensional temperature distributions on a cylindrical fin, optimum dimensions and heat transfer from the fin, the fin efficiency and fin effectiveness of the cylindrical fin when fin base was maintained at a constant temperature. The necessary equations are solved by finite difference method and iteration method using FORTRAN code. The whole investigation was done using different material and different dimensional fins to find out the optimum effectiveness and efficiency for predefined condition.*

Keywords: Pin-fin, Cylindrical fin, 3D fin, Effectiveness of a fin, Efficiency of pin-fin

1. INTRODUCTION

The term extended surface or fin in commonly used to depict an important special case involving heat transfer by conduction within a solid and heat transfer by convection from the boundaries of the solid. Different types of fin such as rectangular fin, triangular fin, trapezoidal fin, parabolic fin, cylindrical fin, pin fin, annular fin etc are commonly used to enhance the heat dissipation rate from primary surfaces to its surrounding fluid medium in order to meet the ever-increasing demand for high performance, light weight and compact heat transfer equipments. Because of many more engineering applications heat transfer characteristics of fins of different geometry have been subject of continued research.

Fins are used to increase the heat transfer from a surface by increasing the effective surface area. However the fin itself represents a conduction resistance to heat transfer from the original surface. For this reason, there is no assurance that the heat transfer rate will be increased through the use of fins. An assessment of this matter may be made by evaluating the fin effectiveness. It is defined as the ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin. In general the use of fins may rarely be justified unless $\varepsilon_f \geq 2$.

Objectives

The main objectives of this study are:

a) To investigate the temperature distribution along the dimension of a cylindrical fin for different thermal conductivity of fin material.
b) To determine the rate of heat transfer through the cylindrical fin.
c) To determine the fin effectiveness and efficiency of the cylindrical fin.
d) To determine the optimum dimension for the cylindrical fin.

2. MATHEMATICAL FORMULATION

2.1 Approximation

The problem is solved, subjected to following assumptions:

Three-Dimensional cylindrical fin, steady state conduction, constant thermal conductivity, homogeneous material, uniform cross section and convection heat transfer coefficient is uniform across the cylindrical fin surface, radiation from the surface is negligible so it is neglected. Fin base and ambient temperature also assumed to be constant.

2.2 Governing Equation

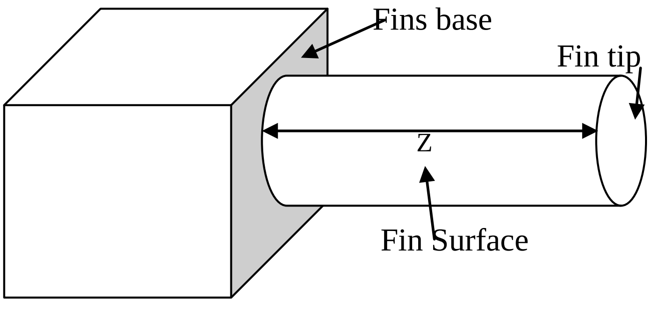
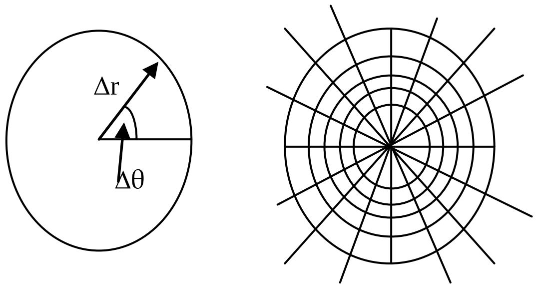


Fig 1: Three dimensional view of the cylindrical fin



(a) (b)

Fig 2: a) Front View of the cylindrical fin b) Front view with grid

The Governing equation for the cylindrical fin is:

$$\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right)+\frac{1}{r^2}\left(\frac{\delta^2 T}{\delta \Phi^2}\right)+\left(\frac{\delta^2 T}{\delta z^2}\right)+\frac{g}{k}=\frac{\delta T}{\alpha \delta t}$$

For steady state condition

$$\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right)+\frac{1}{r^2}\left(\frac{\delta^2 T}{\delta \Phi^2}\right)+\left(\frac{\delta^2 T}{\delta z^2}\right)+\frac{g}{k}=0$$

Now by Finite Difference method we get:

1) $$\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right)=\frac{1}{r}\left(\frac{\delta T}{\delta r}\right)+\frac{\delta^2 T}{\delta r^2}=\frac{1}{r}\left(\frac{T_{(i,j,k)}-T_{(i+1,j,k)}}{\Delta r}\right)+\left(\frac{T_{(i+1,j,k)}+T_{(i-1,j,k)}-2T_{(i,j,k)}}{\Delta r^2}\right)$$

2) $$\frac{1}{r^2}\left(\frac{\delta^2 T}{\delta \Phi^2}\right)=\frac{1}{r^2}\left(\frac{T_{(i,j+1,k)}+T_{(i,j-1,k)}-2T_{(i,j,k)}}{\Delta \Phi^2}\right)$$

3) $$\left(\frac{\delta^2 T}{\delta z^2}\right)=\left(\frac{T_{(i,j,k+1)}+T_{(i,j,k-1)}-2T_{(i,j,k)}}{\Delta Z^2}\right)$$

So the total equation for the conduction in the fin is General conduction equation:

$$T_{i,j,k}=\left(\frac{T_{i+1,j,k}}{r_i\Delta r}+\frac{T_{i+1,j,k}+T_{i-1,j,k}}{\Delta r^2}+\frac{T_{i,j+1,k}+T_{i,j-1,k}}{r_i^2\Delta\theta^2}+\frac{T_{i,j,k+1}+T_{i,j,k-1}}{\Delta z^2}\right)\Big/\left(\frac{1}{r_i\Delta r}+\frac{2}{\Delta r^2}+\frac{2}{r_i^2\Delta\theta^2}+\frac{2}{\Delta z^2}\right)$$

2.2.1 At the tip of the Fin

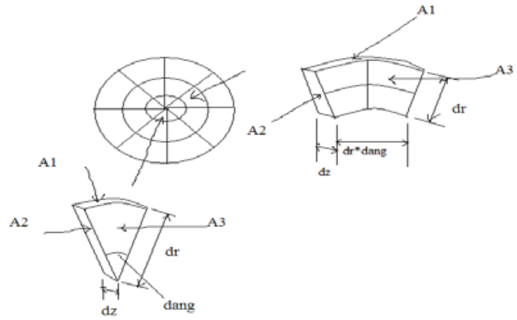


Fig 3: Grid elements of the tip surface of the fin.

At the central grid which is at r1. The grids are triangular so here is the equation of energy balance:

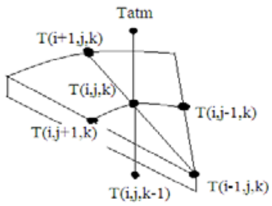


Fig 4: central grid section.

$$-\frac{k\Delta r\Delta\theta\Delta z}{2\Delta r}\left(T_{i,j,k}-T_{i-1,j,k}\right)-\frac{k\Delta r\Delta\theta\Delta z}{2\Delta r}\left(T_{i,j,k}-T_{i+1,j,k}\right)-\frac{k\Delta z\Delta r}{2\Delta r\Delta\theta}\left(T_{i,j,k}-T_{i,j+1,k}\right)-\frac{k\Delta z\Delta r}{2\Delta r\Delta\theta}\left(T_{i,j,k}-T_{i,j-1,k}\right)-\frac{k\Delta\theta\Delta r^2}{2\Delta z}\left(T_{i,j,k}-T_{i,j,k-1}\right)=\frac{h\Delta\theta\Delta r^2}{2\Delta z}\left(T_{i,j,k}-T_{\infty}\right)$$

By simplification:

$$T_{i,j,k}=\frac{\frac{k\Delta\theta\Delta z}{2}\left(T_{i-1,j,k}+T_{i+1,j,k}\right)+\frac{k\Delta z}{2\Delta\theta}\left(T_{i,j-1,k}+T_{i,j+1,k}\right)+\frac{k\Delta\theta\Delta r^2}{2\Delta z}\left(T_{i,j,k-1}\right)+\frac{h\Delta\theta\Delta r^2}{2}T_{\infty}}{\left(\frac{2k\Delta\theta\Delta z}{2}+\frac{2k\Delta z}{2\Delta\theta}+\frac{k\Delta\theta\Delta r^2}{2\Delta z}+\frac{h\Delta\theta\Delta r^2}{2}\right)}$$

The below energy conservation equation is only applied for elements those are after the first circle, which means from the r2 this equation applies.

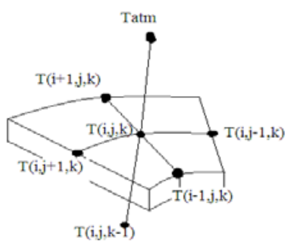


Fig 5: Grid section at 2nd and later circles.

$$-\frac{kr_i\Delta\theta\Delta z}{2\Delta r}\left(T_{i,j,k}-T_{i-1,j,k}\right)-\frac{kr_i\Delta\theta\Delta z}{2\Delta r}\left(T_{i,j,k}-T_{i+1,j,k}\right)-\frac{k\Delta z\Delta r}{2r_i\Delta\theta}$$

$$-\frac{kr_i\Delta\theta\Delta r}{\Delta z}\left(T_{i,j,k}-T_{i,j,k-1}\right)=hr_i\Delta\theta\Delta r(T_{i,j,k}-$$

By simplification:

$$T_{i,j,k} = \frac{\frac{kr_i\Delta\theta\Delta z}{2\Delta r}(T_{i-1,j,k}+T_{i+1,j,k})+\frac{k\Delta z\Delta r}{2r_i\Delta\theta}(T_{i,j-1,k}+T_{i,j+1,k})+\frac{kr_i\Delta\theta\Delta r}{\Delta z}(T_{i,j,k-1})+T_\infty hr_i\Delta\theta\Delta r}{\frac{2kr_i\Delta\theta\Delta z}{2\Delta r}+\frac{2k\Delta z\Delta r}{2r_i\Delta\theta}+\frac{kr_i\Delta\theta\Delta r}{\Delta z}+hr_i\Delta\theta\Delta r}$$

2.2.2 At the Fin Surface

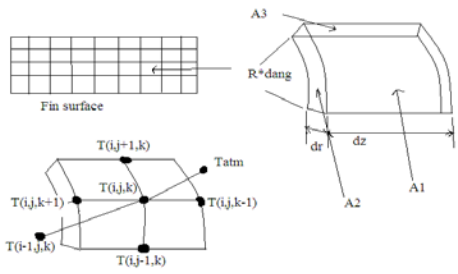


Fig 6: Grids at the surface of the fin.

$$-\frac{kr_L\Delta\theta\Delta z(T_{i,j,k}-T_{i-1,j,k})}{\Delta r}-\frac{k\Delta z\Delta r(2T_{i,j,k}-T_{i,j-1,k}-T_{i,j+1,k})}{2r_L\Delta\theta}-\frac{kr_L\Delta\theta\Delta r(2T_{i,j,k}-T_{i,j,k-1}-T_{i,j,k+1})}{2\Delta z} = hr_L\Delta\theta\Delta z(T_{i,j,k}-T_\infty)$$

By simplification:

$$T_{i,j,k} = \frac{\left(\frac{kr_L\Delta\theta\Delta z(T_{i-1,j,k})}{\Delta r}+\frac{k\Delta z\Delta r(T_{i,j-1,k}+T_{i,j+1,k})}{2r_L\Delta\theta}+\frac{kr_L\Delta\theta\Delta r(T_{i,j,k-1}+T_{i,j,k+1})}{2\Delta z}+T_\infty hr_L\Delta\theta\Delta z\right.}{\frac{kr_L\Delta\theta\Delta z}{\Delta r}+\frac{k\Delta z\Delta r}{r_L\Delta\theta}+\frac{kr_L\Delta\theta\Delta r}{\Delta z}+hr_L\Delta\theta\Delta z}$$

2.2.3 At the edge of the fin

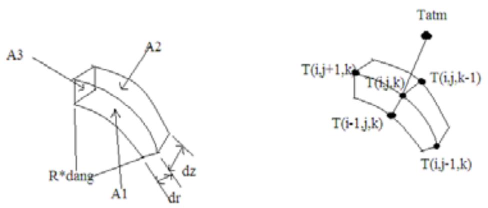


Fig 7: Grid at the edge of the fin.

$$A1 = (\Delta r/2)(r_l\Delta\theta - \Delta r\Delta\theta/4); \quad A2 = \left(\frac{r_l\Delta\theta\Delta z}{2}\right);$$
$$A3 = (\Delta z\Delta r/4)$$

$$-\frac{k*A1(T_{i,j,k}-T_{i,j,k-1})}{\Delta z}-\frac{k*A2(T_{i,j,k}-T_{i-1,j,k})}{\Delta r}-\frac{k*A3(2T_{i,j,k}-T_{i,j-1,k}-T_{i,j+1,k})}{r_L\Delta\theta} = h*A1(T_{i,j,k}-T_\infty)+h*A2(T_{i,j,k}-T_\infty)$$

By simplification:

$$T_{i,j,k} = \frac{\frac{k*A1(T_{i,j,k-1})}{\Delta z}+\frac{k*A2(T_{i-1,j,k})}{\Delta r}+\frac{k*A3(T_{i,j-1,k}+T_{i,j+1,k})}{r_L\Delta\theta}+h*A1*T_\infty+h*A2*T_\infty}{\frac{k*A1}{\Delta z}+\frac{k*A2}{\Delta r}+\frac{2k*A3}{r_L\Delta\theta}+h*A1+h*A2}$$

Fin convective heat transfer from the end:

$$q_f = (hPkA_{cross-section})^{1/2}(T-T_\infty)\frac{\tanh(ml)+(\frac{h}{mk})}{(\frac{h}{mk})\tanh(ml)+1}$$

Convective heat transfer from the fins surface:

$$q_f = \sum h\Delta z(T-T_\infty)$$

Effectiveness of the Fin:

$$\varepsilon_f = \frac{q_f}{hA(T-T_\infty)}$$

Efficiency of the cylindrical fin:

$$\eta_f = \frac{q_f}{hA_{fin}(T-T_\infty)}$$

3. RESULT AND DISCUSSION

The governing three dimentinal differential equation of cylindrical fin was transfomed into linear algebric equations by finite difference methods and these equations were solved by using a program written in FORTRAN language. This code was used to determine the temperatre at each node in the computatinal domain. The material Aluminium, Stainless Steel, Aluminum – Bronze (Alloy), Copper having thermal conductivity (k) 200, 14, 76, 250 and 400 w/m-k respectively were chosen for the analysis of cylindrical fin. The convective coefficient of the surrounding 10 w/sqm-k .The fin base was maintained at a constant base temperature (400°C)673.15K and the surrounding or ambient fluid temperature was considered at (25°C)298.15K.

For testing the Programe a Referance [11] temperature distribution is taken and compared with the result obtained in the figure 8. Similarly another comparison was done for the temperature distribution along the radius. The results are shown in the figure 10.

Variation of effectiveness and effciency due to the variation of material is shown in table 1 and Figure 12 and Figure 13. From the table 1 we can see that copperhas maximum effectiveness and efficiency so it was selected as the material of the fin.

The variation of effectiveness and effciency due to the variation of length and radius is shown in Section 3.1 and Section 3.2 respectively.

At Section 3.3 table 4 shows the changes of effectiveness and efficiency due to change of length and radius simultaneously, thus providing us the optimum dimension of cylindrical fin which is .001m radius and .02m length.

At reference condition: k=206W/m°C, h=17W/sq-m°C, Atmosphere temp=26 °C, Fin base temp=120°C, L=.9m, R=.0127m

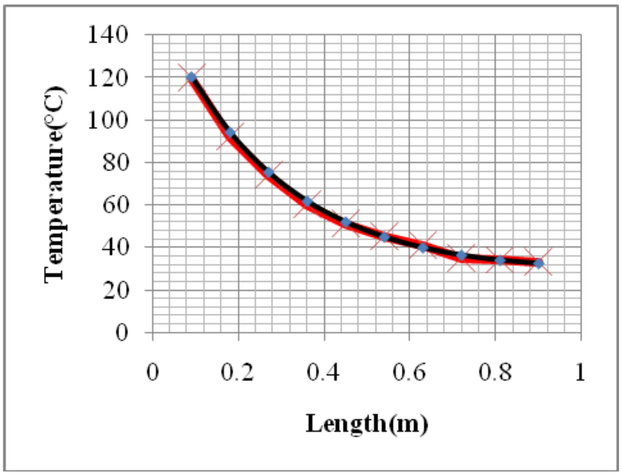


Fig 8: Comparison of result with reference [12](red line) to simulation result (black line)

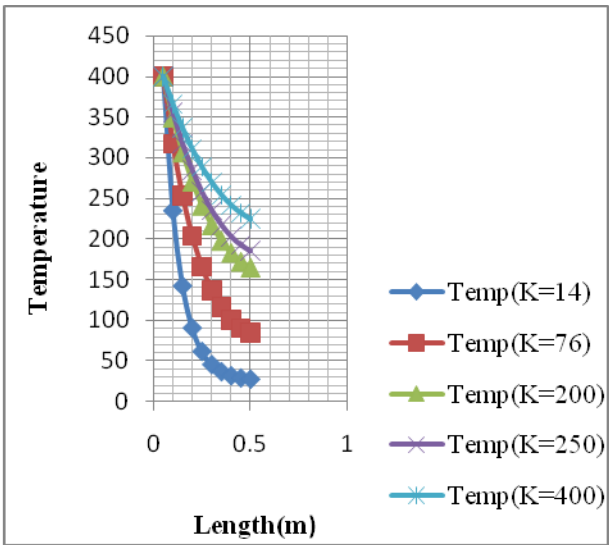


Fig 9: Temperature Distribution along the length of the fin at centre line for different Thermal conductivity (w/m-K)

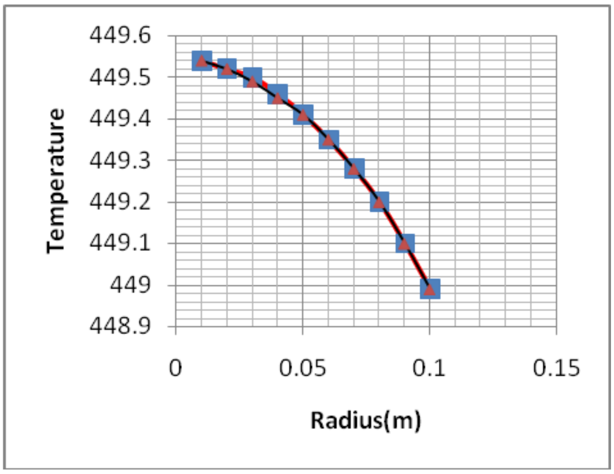


Fig 10: Comparison of Temperature distribution along the radius with the Reference [10] (black line) and simulation result (red line)

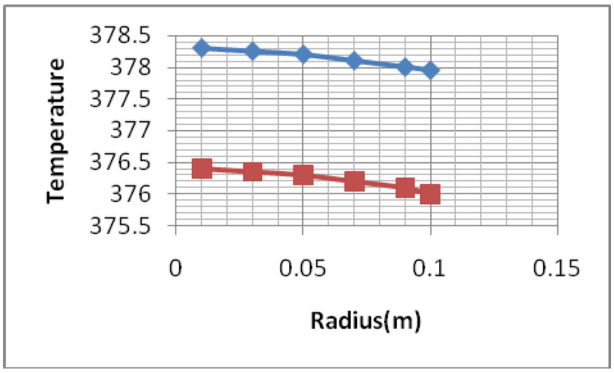


Fig 11: Comparison of temperature distribution of Copper (blue line) and gold (red line)

Table 1: Variation of Efficiency and Effectivness with increasing thermal conductivity for same dimension of the fin:

Thermal conductivity, k	Efficiency(%)	Effectivness
14	38.0184	15.5875
76	72.098	29.5604
200	84.1667	34.508
250	85.98	35.253
400	88.8646	36.4345

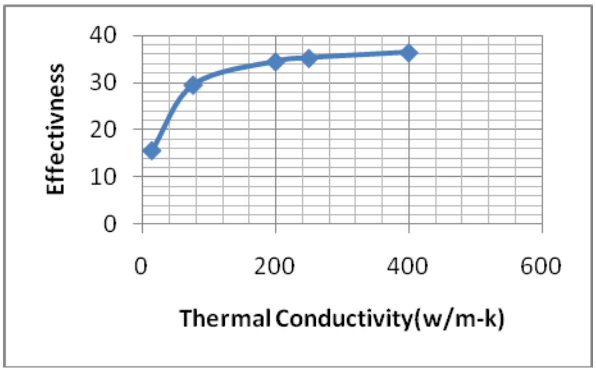


Fig 12: Variation of effectivness with thermal conductivity of material.

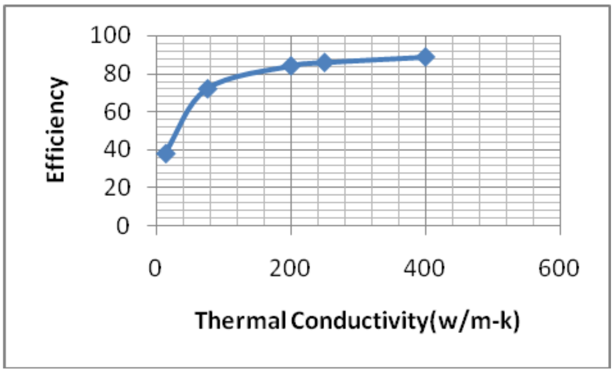


Fig 13: Variation of Efficiency with thermal conductivity of material.

3.1: Variation of Effectiveness and Efficiency with variation of Length for fixed Radius

Table 2: For Copper (k=400 w/m-k)

Length (m)	Radius (m)	Effectiveness	Efficiency(%)
0.02	.01	5.204659	99.6
0.04	.01	9.147603	99.5
0.06	.01	12.92411	99.41621
0.08	.01	16.55999	97.4117
0.1	.01	20.08848	95.65945
0.2	.01	36.4345	88.86464
0.3	.01	50.26515	82.40189
0.4	.01	61.10419	75.43728
0.5	.01	69.24301	68.55745

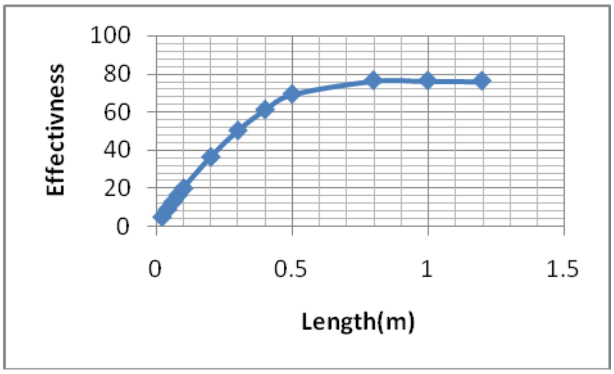


Fig 14: Variation of effectivness with variation of length.(copper)

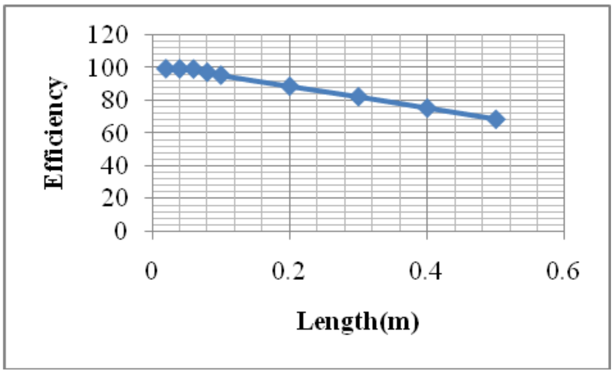


Fig 15: Variation of efficiency with variation of length.(Copper)

3.2:Variation of Effectiveness and Efficiency with variation of Radius for fixed length

Table 3: For Copper (k=400 w/m-k)

Radius(m)	Length(m)	Effectivness	Efficiency(%)
0.008	.5	68.07667	54.0291
0.01	.5	69.24301	68.55745
0.02	.5	40.38728	79.19074
0.04	.5	22.73259	87.43302
0.06	.5	16.20367	91.71889
0.08	.5	12.75182	94.4579
0.1	.5	10.59521	96.32011

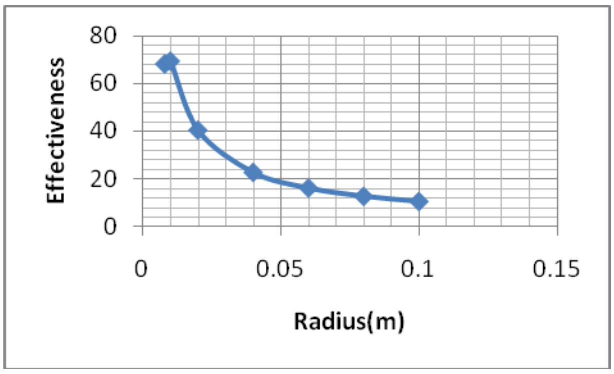


Fig 16 :Variation of Effectivness with variation of radius. (Copper)

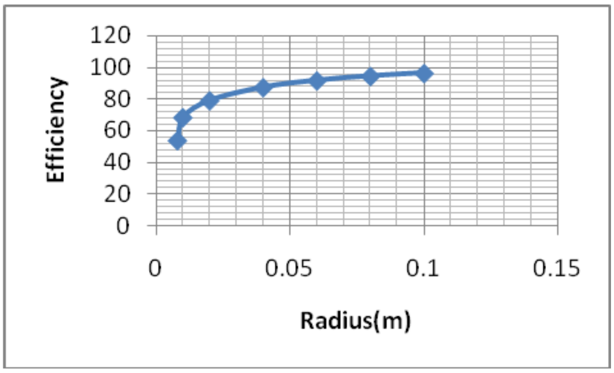


Fig 17: Variation of Efficiency with variation of radius.(Copper)

3.3: Variation of Effectiveness and Efficiency for Different Fin Dimension

Table 4: For Copper(k=400 w/m-k)

Radius (m)	Length (m)	Effectiveness	Efficiency(%)
.02	.2	19.76635	94.12550
.002	.03	29.323790	94.592873
.001	.02	38.379875	93.609459
.003	.04	26.2769	94.977020

From this table the optimum dimensions can be easily found. The one having the maximum effectiveness and maximum effciency. Though the 3rd result has minimum effciency but it has the maximum effectiveness. The daviation of efficiency is not very large so the 3rd result is selected as the optimum dimension.

4. CONCLUSION

From the above information and comparison it's been observed that the optimum dimension for the conditions assumed is a fin having .02m of length and .001m of radius. This fin gave the maximum effectiveness and efficiency for the assumed condition. But the results can vary according to the change of condition, which were assumed to be constant for the purpose of the simplification of the whole process. Material having higher conductivity can be used to get more higher effectiveness and efficiency, but for simplification Copper is selected as the optimum material for the fin.

5. REFERENCES

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5. NOMENCLATURE

Symbol	Meaning	Unit
q_f	Heat transfer rate from the fin	(watt)
A_c	Cross-section area of fin	(sq-m)
K	Thermal conductivity	(W/m-°C)
T(i,j,k)	Temperature At a point	(°C)
r_i	Radius at i-th circle	(m)
Δr	Radius of small element	(m)
$\Delta\theta$	Angle of small element	(radian)
Δz	Length of small element	(m)
T_∞	Ambient Fluid Temperature	(°C)
r_l	Maximum radius	(m)
L	Maximum length of fin	(m)
ε_f	Fin effectiveness	Dimensionless
η_f	Fin efficiency	Dimensionless
A_{fin}	Fin Surface area	(sq-m)
q_{max}	Maximum heat transfer if the fin surface is at base temperature	(watt)
i	Index along radial direction (r-axis)	Dimensionless
j	Index along angular direction (θ-axis)	Dimensionless
k	Index along length direction (z-axis)	Dimensionless
p	Fin perimeter (m)	(m)
q_b	Heat transfer from fin base	(watt)