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DYNAMIC MODELING AND TRAJECTORY TRACKING OF WHEELED SKID-STEERED UNMANNED GROUND VEHICLE WITH SLIP

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Abstract- *Mobile robotic applications require a wide range of steering activity in order to achieve sufficient maneuverability on rough terrain. There have been wide applications of mobile robots using skid-steering technique for terrain exploration and navigation due to the simplicity and high reliability of the mechanism. This paper presents the mathematical modeling of a wheeled skid-steered Unmanned Ground Vehicle (UGV) under the slip condition. The mathematical model is developed based on the dynamics working on the UGV during its motion while following a given trajectory. In order to do this, kinematic and dynamic modeling of the UGV is first presented under the consideration of longitudinal slip. A friction model is then developed to analyze the wheel/ground interaction as well as the UGV motion. The performance of the proposed dynamic model of the UGV is analyzed by demonstrating the numerical results in the simulation and the path following response.*

Keywords: Mobile robot dynamics, Skid-steer, UGV, Slip, Trajectory tracking.

1. INTRODUCTION

In these days, mobile robots are used extensively for their ability to navigate and perform tasks in unstructured environments. These robots need to be tested under different conditions before they are put into work. It is necessarily required that one models the actual vehicle and simulates test conditions similar to those robots that might encounter while developing or testing the robot. Vehicle dynamics is concerned with the movements of vehicles, e.g., automobiles such as trucks, buses, and special purpose vehicles. It accommodates all forms of conveyance using rubber tired vehicles, track laying vehicles, trains etc. Previously, several researches have been focused on the description of kinematic models of mobile robots typically considering wheeled platforms with nonholonomic constraints while assuming perfect rolling. The control of the robot only at the kinematic level is not sufficient and, in general, demands the use of a dynamic model which is a really challenging task to the researchers.

In recent days, the uses of skid-steering mobile robots or vehicles have been very popular in many applications for their greatly simple navigation capability and mechanical robustness of the steering system. Despite of having so many advantages, such as high maneuverability, high power, and ability of working in hard environmental conditions with simple steering mechanism; it is quite challenging to get dynamic models so accurately and trajectory tracking control system for such mobile robots because of the varying tire/ground interactions and over constrained contact.

Fig.1: Schematic of a four wheeled skid-steered UGV.

A critical role played by wheel slip both longitudinally and laterally makes the kinematic and dynamic modeling of the skid-steered robots very complicated. Zhang *et al.* [1] derive a simplified dynamic model which is adequate for control design and treat the remaining terms as model uncertainty. In [2], Shiller *et al.* determine the nominal track forces required to follow a specified path at desired speeds and compute vehicle orientations along the path that are consistent with the nonholonomic constraint. In [3], a trajectory tracking control problem for a four-wheel differentially driven mobile robot moving is considered by Caracciolo *et al.* on an outdoor terrain. In [4], a path tracking control algorithm for tracked surface drilling machines is presented by Ahmadi *et al.* in which the general dynamic model of vehicle is simulated including track-soil interactions. Wang *et al.* [5] develop a trajectory planning algorithm for a four-wheel-steering vehicle based on vehicle kinematics in which the flexibility offered by the steering is utilized fully in the trajectory planning. In [6], a mathematical model of a 4-wheel skid-steering mobile robot is presented by Kozłowski *et al.* in a systematic way where the robot is considered as a subsystem consisting of kinematic, dynamic and drives levels. In [7], Jingang *et al.* again present localization and slip estimation scheme for a skid-steered mobile robot using low-cost inertial measurement units (IMU). They again present an adaptive trajectory control design for a skid-steered wheeled mobile robot with kinematic and dynamic modeling of the robot [8]. Later, Wang *et al.* in [9] aim to give a general and unifying presentation on modeling of wheeled mobile robots in the presence of wheel skidding and slipping from the perspective of control design. Although it is challenging to dynamically analyze the tracking of wheeled mobile robot, the focus of the paper is on dynamic modeling of wheeled skid-steered unmanned ground vehicle (UGV) and trajectory tracking.

We discussed primarily the kinematic as well as the dynamic modeling of a wheeled skid-steered UGV in section 2 of this paper. Section 3 describes the state-space representation of the UGV dynamic model where the dimensional and input parameters are included. The responses of the trajectory tracking are demonstrated in section 4. Finally, we discuss the effectiveness of the model and future research directions as paper conclusion in section 5.

2. DYNAMIC MODEL

Modeling Assumptions

1. Vehicle is rigid and moving on a horizontal plane.
2. Vehicle speed is very low.
3. Lateral force of tire is the function of its vertical load.
4. Vehicle rotation is counterclockwise.
5. Wheel actuation is equal on each side.

Figure 2 represents the schematic of the wheeled skid-steered UGV according to which a fixed reference or global frame is defined as $F(X, Y)$ and a moving body or local frame as $f(x, y)$. The center of mass is located at distances a and b from front and rear wheels respectively ($a < b$). Wheelbase is $2t$ and symmetric on each side.

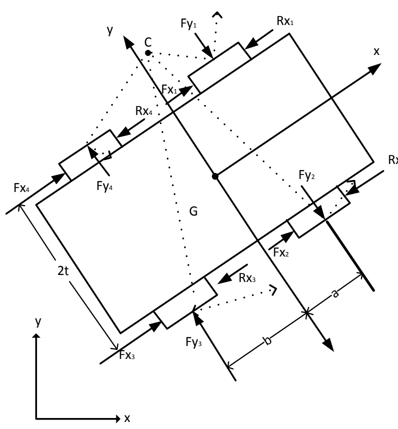


Fig.2: Top view schematic of a wheeled skid-steered UGV on a flat surface.

Let $\dot{x}, \dot{y}, \dot{\theta}$ be the longitudinal, lateral and angular velocity of the vehicle in frame f respectively. The absolute velocities in frame F are

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \dot{x}\cos\theta - \dot{y}\sin\theta \\ \dot{x}\sin\theta + \dot{y}\cos\theta \end{bmatrix} = R^T(\theta)\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

Differentiation with respect to time

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} = R^T(\theta)\begin{bmatrix} \ddot{x} - \dot{y}\dot{\theta} \\ \ddot{y} + \dot{x}\dot{\theta} \end{bmatrix} = R^T(\theta)\begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

where, a_x and a_y are the absolute accelerations in frame f. The vehicle motion is a pure rotation around the instantaneous center of rotation C at each instant. Its coordinates are

$$\begin{bmatrix} x_c \\ y_c \end{bmatrix} = \begin{bmatrix} -\dot{y} / \dot{\theta} \\ \dot{x} / \dot{\theta} \end{bmatrix}$$

During the straight line motion, both the angular velocity $\dot{\theta}$ and the lateral velocity $\dot{y}$ are vanished, and the ICR goes to infinity along the y-axis. Only for the circular motion or motions besides the one following straight line, there should be a finite value of the ICR. It shifts forward by an amount $|x_c|$ in case of curved path. When $\dot{y} = 0$, there is no lateral skidding.

Longitudinal velocity $\dot{x}_i$ and lateral (skidding) velocity $\dot{y}_i$ of each wheel (i =1, 2, 3, 4) are given by

$$\begin{aligned} &\dot{x}_1 = \dot{x}_4 = \dot{x} - t\dot{\theta} \text{ (left)} \\ &\dot{x}_2 = \dot{x}_3 = \dot{x} + t\dot{\theta} \text{ (right)} \\ &\dot{y}_1 = \dot{y}_2 = \dot{y} + a\dot{\theta} \text{ (front)} \\ &\dot{y}_3 = \dot{y}_4 = \dot{y} - b\dot{\theta} \text{ (rear)} \end{aligned} \qquad (i = 1, 2, 3, 4) \qquad (1)$$

In the free-body diagram, forces and velocities are shown with the vehicle having instantaneous positive velocity components $\dot{x}$ and $\dot{\theta}$ and negative velocity $\dot{\theta}$. Wheels develop tractive forces F_{xi} and are subjected to longitudinal resistive forces R_{xi}, for i = 1, 2, 3, 4. Assume that wheel actuation is equal on each side so as to reduce longitudinal slip. Thus, it will always be $F_{x4} = F_{x1}$ and $F_{x3} = F_{x2}$. Lateral forces F_{yi} act on the wheels as a consequence of lateral skidding. A resistive moment M_r around the center of mass is induced in general by the F_{yi} and R_{xi} forces. Let, mass of the vehicle = m and inertia about its center of mass = I. Equations of motion in body frame f are

$$\begin{aligned} ma_x &= 2F_{x1} + 2F_{x2} - R_x \\ ma_y &= -F_y \\ I\ddot{\theta} &= -tF_{x1} - tF_{x4} + tF_{x2} + tF_{x3} + M_r \\ &= 2t(F_{x2} - F_{x1}) + M_r \end{aligned} \qquad (2)$$

There should be consideration of how the vehicle gravitational load mg is shared among the wheels.

Introducing a Coulomb friction model for the wheel/ground contact, we have

$$F_{z1} = F_{z2} = \frac{b}{a+b}\frac{mg}{2}$$
$$F_{z3} = F_{z4} = \frac{a}{a+b}\frac{mg}{2}$$

The lateral load transfer due to centrifugal forces on curved paths at low speed can be neglected. In case of hard ground, we can assume that the contact patch between wheel and ground is rectangular and that the tire vertical load produces a uniform pressure distribution. In this condition, longitudinal resistive force,

$$R_{xi} = \sum_{i=1}^{4} R_{xi} = f_r \frac{mg}{2}[\mathrm{sgn}(\dot{x}_1) + \mathrm{sgn}(\dot{x}_2)] \quad (3)$$

Lateral friction coefficient is introduced considering the Coulomb friction model for the wheel ground contact. Therefore, the lateral force is

$$F_{yi} = \sum_{i=1}^{4} F_{yi} = \mu \frac{mg}{a+b}[b\,\mathrm{sgn}(\dot{y}_1) + a\,\mathrm{sgn}(\dot{y}_3)] \quad (4)$$

Resistive moment is

$$\begin{aligned} M_r &= a(F_{y1} + F_{y2}) - b(F_{y3} + F_{y4}) \\ &\quad + t[(R_{x2} + R_{x3}) - (R_{x1} + R_{x4})] \\ &= -\mu \frac{abmg}{a+b}[\mathrm{sgn}(\dot{y}_1) - \mathrm{sgn}(\dot{y}_3)] \\ &\quad + f_r \frac{tmg}{2}[\mathrm{sgn}(\dot{x}_2) - \mathrm{sgn}(\dot{x}_1)] \end{aligned} \quad (5)$$

Now a generalized coordinates $q = (X, Y, \theta)$ is introduced to rewrite the dynamic model in frame F as

$$M\ddot{q} + c(q, \dot{q}) = E(q)\tau \quad (6)$$

where, $M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}; c(q,\dot{q}) = \begin{bmatrix} R_x \cos\theta - F_y \sin\theta \\ R_x \sin\theta + F_y \cos\theta \\ -M_r \end{bmatrix}$

and $E(q) = \begin{bmatrix} \cos\theta / r & \cos\theta / r \\ \sin\theta / r & \sin\theta / r \\ -t / r & t / r \end{bmatrix}$

$r =$ wheel radius

$\tau_1, \tau_2 =$ torques of left and right wheel motors respectively.

3. MATLAB SIMULATION

The three degree-of-freedom vehicle model equations of motion (2) can be rewritten in first order differential equation form to enable using first-order numerical integration methods. Inputs into the vehicle are torques provided by LHS and RHS side motors. The third order Runge-Kutta integration routine is used to integrate vehicle equations of motion. The Matlab command ode45 is used to solve simultaneous first order differential equations.

State space representation of dynamic equations (2) is as follows

$$\begin{aligned} X_1 &= \dot{\theta} \\ X_2 &= \dot{x} \\ X_3 &= \dot{y} \\ X_4 &= X \\ X_5 &= Y \\ X_6 &= \theta \\ \dot{X}_1 &= \frac{2t(F_{x2} - F_{x1}) + M_r}{I} \\ \dot{X}_2 &= \frac{2(F_{x1} + F_{x2}) - R_x}{I} - X_3 X_1 \\ \dot{X}_3 &= -\frac{F_y}{m} + X_2 X_1 \\ \dot{X}_4 &= X_2 \cos X_6 - X_3 \sin X_6 \\ \dot{X}_5 &= X_2 \sin X_6 + X_3 \cos X_6 \\ \dot{X}_6 &= X_1 \end{aligned}$$

The input torques supplied by the motors are $\tau_1 = 2rF_{x1}$ and $\tau_2 = 2rF_{x2}$. The above equations can be solved in Matlab using the command ode45. Vehicle parameters used for simulation are:
a = 0.37 m, b = 0.55 m, $2t$ = 0.63 m, r = 0.2, m = 116 kg and I = 20 kg-m^2

4. RESULTS

Case I: If the input torques to both motors is same, the robot should travel along a straight path as shown in the plot of Y vs. X.

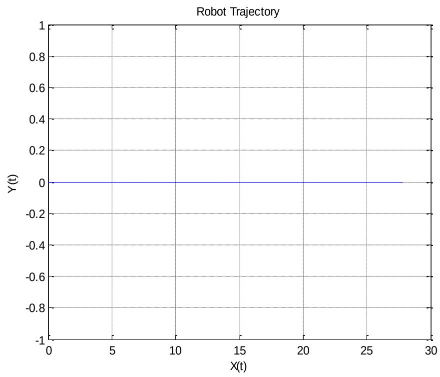


Fig.3: UGV trajectory (case I).

Case II: If the right wheel torque is larger than the left wheel torque, the UGV follows the motion as shown in fig. 4.

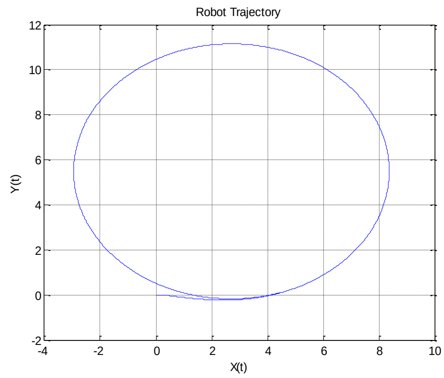


Fig.4: UGV trajectory (case II).

It can be clearly seen from the above plot that the UGV forms a circular trajectory after some time from the beginning. Angular acceleration decreases with time and remains at steady state value after some time as the torque difference between motors increases with time.

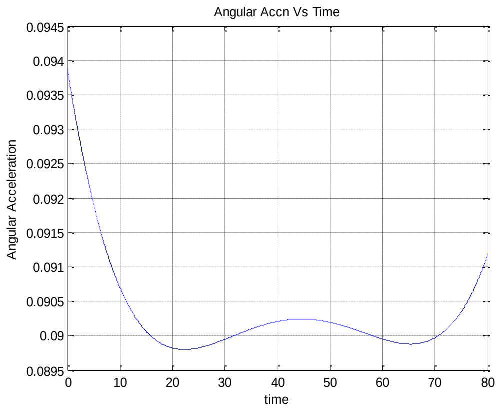
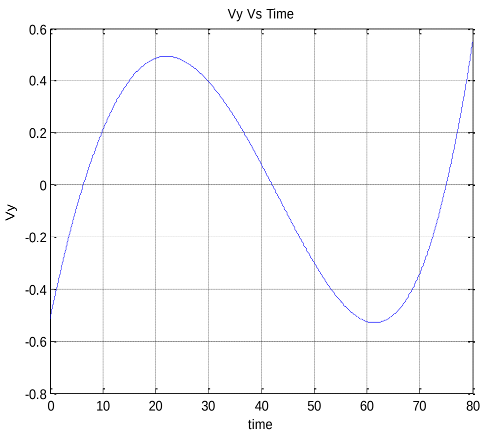
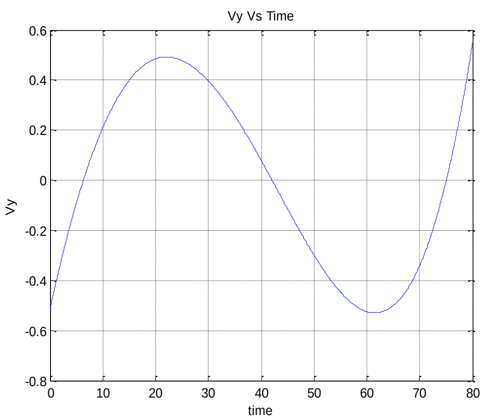


Fig.5: Plot showing angular Acceleration vs. Time.

V_x vs. Time & V_y vs. Time



(a)



(b)

Fig.6: Plot showing (a) V_x vs. Time and (b) V_y vs. Time

Lateral vs. Longitudinal Velocity

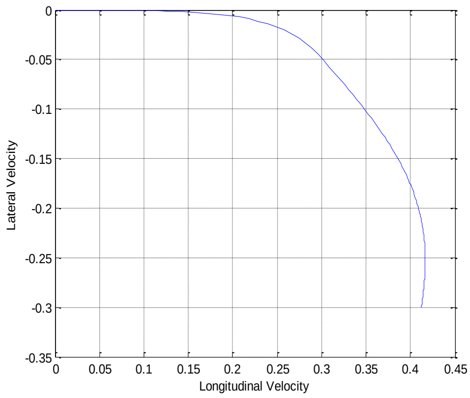


Fig.7: Plot showing longitudinal vs. lateral velocity.

5. CONCLUSION

In this paper, a model of wheeled skid-steered UGV was developed analytically and then dynamic analysis was performed in Matlab Simulink. In this analysis, the UGV dynamics was adopted to follow a given trajectory where longitudinal slip was included that made the model distinct and a bit complex rather than the other models excluding slip. The relationship between the longitudinal and lateral friction forces at each wheel was obtained through the rigid body kinematics of the robot frame and wheels on each side. Despite of having such complexity, the UGV is capable of following a good shaped trajectory as required. Future works could involve nonlinear slip-friction model in order to take such dynamic models close to reality. Dynamic feedback linearization based control technique could be proposed to build a good adaptive controller for better trajectory response in case of experiments on rough terrain.

6. REFERENCES

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